Low-lying structure and shape evolution in neutron-rich Se isotopes

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Neutron-rich 58,90,92,94Se isotopes were studied via in-beam γ-ray spectroscopy after neutron removal reactions at intermediate energies at the Radioactive Isotope Beam Factory. Based on γ-γ coincidence analysis, low-lying excitation level schemes are proposed for these nuclei, including the 2+ γ states at remarkably low energies. The low-lying 2+ γ states, along with other features, indicate triaxiality in these nuclei. The experimental results are in good overall agreement with self-consistent beyond-mean-field calculations based on the Gogny D1S interaction, which suggests both triaxial degree of freedom and shape coexistence playing important roles in the description of intrinsic deformations in neutron-rich Se isotopes.

Atomic nuclei are complex systems composed of interacting protons and neutrons. Nevertheless, their low-lying energy levels can often be simply described by rotations and vibrations around intrinsic shapes. Due to the underlying shell structure, governed by the effective nuclear interaction, such intrinsic shapes evolve with the number of nucleons in the system. Equilibrium shapes are frequently associated with the occurrence of shell gaps in Nilsson-like single-particle levels around the Fermi level. When moving from closed-shell to open-shell nuclei, transitions from spherical to deformed shapes can be observed along isotopic and isotonic chains. Generally, ground-state deformations evolve smoothly as a function of nucleon number. In some regions of the nuclear chart, however, rapid shape transitions occur, indicating coexisting different intrinsic shapes competing at low excitation energy [1,2].

The neutron-rich nuclei around N ≈ 60 have been identified as a region of shape coexistence and shape transitions [2–6]. The Zr and Sr isotopes in this region exhibit sudden changes from almost spherical to highly deformed ground states at N = 60, which belong to the most drastic shape transitions in the nuclear chart and were interpreted as intruders of coexisting deformed shapes [2]. Recent experimental studies of these isotopes have confirmed the competition of low-lying spherical and deformed configurations in 96,98Zr [7] and established shape coexistence in 94,96Zr,98Sr [8,9]. Conversely, for the neutron-rich Kr isotopes, although evidence for a gradual increase of collectivity has been found up to N = 60 [10–12], yielding no sudden change from spherical to strong deformation shapes, a possible prolate-oblate shape coexistence has been proposed for 96Kr and heavier isotopes [5,12].

Experimental data and theoretical studies for the neutron-rich Se isotopes are scarce. The heaviest isotope for which mass and β-decay half-life are established is 50Se [13,14], while low-lying excited states have been observed only up
to \(N = 53, 54\) \([15–17]\), both far away from the rapid onset of ground-state deformation found at \(N = 60\) in Sr and Zr nuclides. The exit reaction \(\gamma\)-ray of \(98\)Se corresponding to the \(2^+\) level in \(98\)Se is of interest. However, \(\gamma\) spectra for subshell signatures from half-life measurements provided no support for the existence of an \(N = 56\) subshell \([14]\). In addition, a recent experimental study of \(98\)Se by \(\gamma\)-ray spectroscopy reported a possible \(3^+\) level, indicating an onset of \(\gamma\) collectivity in \(98\)Se \([18]\). Meanwhile, the \(B(E2)\) measurement and comparison with shell-model calculations suggest a triaxial shape in \(96\)Se \([19]\). Moving to more neutron-rich \(\text{Se}\) isotopes, it is expected that nonaxial degrees of freedom play an important role in the description of deformation shapes \([20]\). Furthermore, systematic theoretical calculations in this region \([4, 21]\) predict close-lying prolate and oblate minima in potential energy curves, originating from the down-sloping \(v_{112}\) orbitals, which also suggest shape coexistence and ground-state shape transition in the \(\text{Se}\) isotopic chain.

The goal of this paper is twofold: (i) the identification of low-lying excited states of the very neutron-rich even isotopes \(98–104\)Se in order to assess their structures up to \(N = 60\); and (ii) the comparison of the experimental data with symmetry-conserving configuration mixing (SCCM) calculations that indicate that this region of the isotopic chain shows evidences of sudden prolate-oblate shape transitions and shape coexistence.

The experiment was carried out at the Radioactive Isotope Beam Factory, operated by the RIKEN Nishina Center and the Center for Nuclear Study of the University of Tokyo. A \(^{238}\text{U}\) primary beam, accelerated to an energy of \(345\ MeV/u\), was provided with an average intensity of \(50\ p\text{mA}\). Secondary radioactive isotope beams were produced by in-flight fission of the primary beam in a 3-mm-thick Be production target placed at the entrance of the fragment separator BigRIPS \([22]\). The isotopes of interest were selected and separated with the \(\beta\)-\(\Delta\)E-\(\beta\)-\(p\) method and identified on an event-by-event basis by a TOF-\(\beta\)-\(\Delta\)E measurement \([23]\). Two BigRIPS settings were applied and centered on \(^{90}\text{As}\) and \(^{90}\text{Br}\), respectively. The average intensities of the \(^{90}\text{Se}\) and \(^{91, 94}\text{Br}\) isotopes, which were used to produce the \(\text{Se}\) isotopes of interest, were measured to be \(618, 1280, 182,\) and \(50\) particles per second, respectively, and impinged on a \(99(1)\)-mm-thick \((725(7)\) mg/cm\(^2\)) liquid hydrogen reaction target. Their energies in front of the reaction target were \(\sim \) \(270\ MeV/u\) and decreased to \(\sim \) \(180\ MeV/u\) at the exit of the reaction target. Reaction residues were collected and unambiguously identified by the ZeroDegree spectrometer \([22]\), employing a similar identification method as described for BigRIPS.

Surrounding the liquid hydrogen target was a 300-mm-long cylindrical time projection chamber (TPC) of the MINOS ensemble \([24]\). The usage of the thick liquid hydrogen target required the employment of the TPC to track the protons for \((p, 2p)\) and other reactions and to reconstruct their vertex position for the Doppler correction. An efficiency of \(>91\%\) for good vertex reconstruction of the \(^{91}\text{Br}(p, 2p)^{90}\text{Se}\) channel was obtained with a spatial resolution of \(\sim 5\) mm FWHM. Examples of reconstructed vertices from two proton trajectories for \((p, 2p)\) knock-out reactions are illustrated in Refs. \([24, 25]\). In the case of \((p, pn)\) channels, the beam and scattered proton trajectories were used to determine the reaction vertex. Deexcitation \(\gamma\) rays were measured by the DALI2 detector array \([26, 27]\), which consisted of \(186\) NaI(Tl) scintillators. A full-energy peak detection efficiency of \(35\%\) (23\%), including add-back, was simulated for 500-keV \((1\text{-MeV})\) \(\gamma\) rays emitted from the target center at a beam energy of \(250\ MeV/u\) with the GEANT4 framework \([28]\). Energy calibrations were performed for each BigRIPS-ZeroDegree setting using \(^{50}\text{Co},\ ^{137}\text{Cs},\ ^{88}\text{Y},\) and \(^{133}\text{Ba}\) sources, resulting in a calibration error of 2 keV in the range of 350–1300 keV and an energy resolution of 9\% (6\%) FWHM at 662 keV (1.33 MeV), consistent with Refs. \([26, 27]\).

The Doppler-corrected spectra for \(^{88–94}\text{Se}\) are shown in Fig. 1, assuming all \(\gamma\) rays were emitted from the reaction vertex reconstructed by MINOS. However, the lifetimes of the excited states influenced the Doppler correction. This effect was considered when obtaining the DALI2 response functions from GEANT4 simulations by assuming lifetimes based on the theoretical approach discussed later and included in the errors for the energy determinations. Furthermore, the validity of the Doppler correction was verified using the well-known \(4^+_1 \to 2^+_2\) and \(2^+_1 \to 0^+_g.s.\) transitions of \(^{84}\text{Kr}\) \([11, 29]\) following \((p, pn)\) reactions, which yielded less than 1 keV deviation to the literature values. Each spectrum was fitted with DALI2 response functions added on top of a double-exponential background. \(\gamma\)-\(\gamma\) coincidences were used to establish level schemes for decays observed in the single spectra. Examples of this analysis as well as the established level schemes are given in the insets of Fig. 1. However, the current measurement was not sensitive to spin parities. Thus, their assignments were based on the systematics of the isotopic chain and the comparison with other studies of knock-out reactions \([25, 31, 32]\).

For \(^{88}\text{Se}\), the previously reported \(\gamma\)-ray line at 886 keV assigned to the \(2^+_1 \to 0^+_g.s.\) transition \([16]\) was not observed. Instead, the most intense \(\gamma\)-ray transition after efficiency correction was found at 580(8) keV, which was assigned to the \(2^+_1 \to 0^+_g.s.\) decay as in other nuclei studies \([25, 31, 32]\). Toward the high-energy side, a tail was observed and fitted by another transition at 646(15) keV. Two other transitions were observed at 971(10) and 1232(13) keV. The \(\gamma\)-\(\gamma\) analysis showed that the 646- and 971-keV transitions were in coincidence with the \(2^+_1\) state, while, within the uncertainties, the 1232-keV transition was in good agreement with the sum of the 580- and 646-keV transitions. Therefore, the latter was assigned to the \(2^+_2 \to 2^+_1\) transition, while the former were assigned to the \(2^+_1 \to 0^+_g.s.\) transition. For \(^{90}\text{Se}\), the most intense transition, here at 548 keV, was assigned to the \(2^+_1 \to 0^+_g.s.\) decay. The \(\gamma\)-\(\gamma\) analysis revealed only the 960-keV peak was not in coincidence with the 548-keV transition [see Fig 1(b) inset], while it matched the sum of the 419- and 548-keV decays within errors. Thus, we assigned the 960- and 419-keV transitions to the \(2^+_2 \to 0^+_g.s.\) and \(2^+_2 \to 2^+_1\) transitions, respectively. The 691- and 1075-keV transitions were placed on top of the \(2^+_1\) state in the proposed level
In a previous isomer study, γ-ray lines for $^{92}\text{Se}$ were reported at 503, 539, and 898 keV, but no level scheme was established [33]. In the present work, seven transitions were found at 429(7), 539(9), 624(13), 715(7), 898(15), 958(22), and 1061(14) keV, with the 539-keV transition assigned to the $2^+_1 \rightarrow 0^+_{s.}$ and the 715-keV transition assigned to the $4^+_2 \rightarrow 2^+_1$, based on the relative intensities. It is noted that owing to the accurately known transition at 898 keV, the peak at 958(22) keV could be identified, which was revealed as a doublet with the 898-keV transition in the form of a high-energy tail. Note that lifetime effects may only lead to tails toward the lower energies and calculations using the theoretical $B(E2)$ values suggest all excited states should have lifetimes shorter than 50 ps. Based on the γ-γ analysis and the sum of transition energies, we assigned the 429- and 958-keV γ-ray decay from the $2^+_1$ state at 968(11) keV, and the 624- and 1061-keV γ-ray decay from a level at 1600(17) keV, which was tentatively assigned to be the (3,4$^+$) from comparison to the level scheme of $^{90}\text{Se}$ [18]. The 898-keV transition could not be placed in the proposed level scheme. It is further noted that the coincidence spectrum features a reduced self-coincidence with the $2^+_1$ decay due to Compton events originating from high-energy transitions.

For the most exotic isotope studied here, $^{94}\text{Se}$, two clear transitions were seen at 475(10) and 640(7) keV and in coincidence with each other. They were assigned to the $2^+_1 \rightarrow 0^+_{s.}$ and $4^+_2 \rightarrow 2^+_1$ transitions based on relative intensities. In addition, three peak-like structures were observed at 830(30), 1290(30), and 1580(30) keV. Standard significance tests [34] yielded 3.6$\sigma$, 3.0$\sigma$, and 3.7$\sigma$, respectively, for these candidate peaks (we required a significance of at least 5$\sigma$ for an assignment of a new transition). The 1290 keV matches well the sum of 475 and 830 keV within errors, thus a candidate $2^+_1$ level at 1290(30) keV was placed in the level scheme. Note that a γ-γ analysis could not be performed for these candidate peaks due to limited statistics.

The systematics of $E(2^+_1)$, $R_{4/2}$ ratio, and $E(2^+_2)$ for Se in comparison with Zr, Sr, and Kr isotopes are shown in Fig. 2 from $N = 50$ to 60. A gradual decrease of $E(2^+_1)$ for the Se isotopes is apparent, yielding no indication for a subshell closure at $N = 56$ nor a sharp rise of deformation at $N = 60$. The $R_{4/2}$ ratio increases from $N = 50$ to 54, followed by a drop at $N = 56$, then stays roughly constant at values around 2.3 until $N = 60$. When comparing with other isotopic chains, both the $E(2^+_1)$ and $R_{4/2}$ pattern are at variance with the Zr and Sr isotopes and strongly resemble the trend of the Kr isotopes [11,12,29]. More interestingly, the assigned $2^+_1$ levels keep decreasing until $N = 56,58$, then show a possible increase at $N = 60$, especially for $N = 56,58$, the $E(2^+_1)$ are among the lowest in this mass region [see Fig 2(c)] and thus indicate γ-soft or triaxial features in these nuclei. Further, the $B(E2;2^+_1 \rightarrow 2^+_1)/B(E2;2^+_2 \rightarrow 0^+_{s.})$ ratios calculated from the experimental branching ratios and level energies, assuming negligible $B(M1)$ contributions, give values of 24(4), 20(4), 161(57), and 9(5) for $^{88,90,92,94}\text{Se}$, respectively. These large ratios are consistent with the O(6) limit in the interacting boson approximation (IBA) model [35].

In the following, the experimental results are compared with a symmetry-conserving configuration mixing (SCCM)

scheme, with the 1239-keV state assigned to the $4^+_2$ and the 1623-keV state tentatively assigned to a (3,4$^+$) state, based on the comparison to $^{92}\text{Se}$ discussed below. A (3,4$^+$) → $2^+_1$ transition remained unobserved, possibly because it coincided energetically with the $4^+_1 \rightarrow 2^+_1$ transition.
calculation with Gogny D1S effective interaction \cite{36,37}. In this framework, each individual nuclear state is defined as the linear combination of multiple intrinsic many-body states with different quadrupole (axial and triaxial) shapes. Further details are described in Refs. \cite{5,38–40}. In addition, only time-reversal (static) and parity symmetric intrinsic shapes were considered, i.e., cranked or octupole deformed states were not included. Therefore, a systematic stretching of the levels with respect to the experimental values is expected \cite{41} and negative parity bands cannot be described. We note that in the $(p,2p)$ reactions studied in this paper, direct population of negative parity states requires proton knockout from the $g_{9/2}$ orbital. This orbital is expected to be only weakly occupied in the ground states of the Br isotopes, as inferred from the recent calculations for the Zr isotopes \cite{6}. Therefore, the present theory is considered sufficient to describe all observed levels.

In Fig. 3, the theoretical predicted level energies are compared with experimental results for \textit{86–94}Se. Low-lying 2\textsuperscript{+} states below the 4\textsuperscript{+} levels are well reproduced for 86–92Se and an increase of $E(2\textsuperscript{+})$ is also predicted for 94Se. As remarked above, a general stretching of the theoretical predictions is observed, mainly due to the privileged exploration of the ground-state energy in the present variational process without cranking states \cite{41}. Despite the stretching, the present SCCM calculation describes rather well the trends of the experimental energies for the 2\textsuperscript{+}, 4\textsuperscript{+}, and 2\textsuperscript{+} states, although some differences are found, e.g., the decrease of the $E(2\textsuperscript{+})$ from 88Se to 90Se and of the $E(2\textsuperscript{+})$ from 90Se to 92Se instead of the flat experimental trend. The present SCCM calculations also yield large $B(E2;2\textsuperscript{+} \rightarrow 2\textsuperscript{+})/B(E2;2\textsuperscript{+} \rightarrow 0\textsuperscript{+})$ ratios of 49, 11, 41, and 5 for 88,90,92,94Se, respectively. These calculated ratios are not as extensive as measured, yet the systematic trend is correctly obtained. Further, calculated $B(E2;2\textsuperscript{+} \rightarrow 0\textsuperscript{+})$ values can be compared to available experimental data. The calculations yield values of 411 e\textsuperscript{2} fm\textsuperscript{4} and 546 e\textsuperscript{2} fm\textsuperscript{4} compared to experimental values of 210(30) e\textsuperscript{2} fm\textsuperscript{4} and 438(5) e\textsuperscript{2} fm\textsuperscript{4} for 84,86Se \cite{19}, respectively. The discrepancy for the semi-magic 84Se is expected due to the relevance of quasiparticle excitations not taken into account in the present SCCM framework. In addition, the present SCCM calculation favors a 4\textsuperscript{+} assignment for the (3,4\textsuperscript{+}) levels in 90–94Se. Predicted 3\textsuperscript{+} states, which belong to the $\gamma$ bands, lie about 1 MeV higher in energy [see Fig. 3(b)].

To gain insight into the intrinsic deformation, we display in Fig. 4 the potential energy surfaces (PESs) for 86–94Se together with the collective wave functions (c.w.f.) for the 0\textsuperscript{+}, 2\textsuperscript{+}, and 2\textsuperscript{+} states. All c.w.f. have maxima in probabilities at $\beta_2 \sim 0.2–0.3$ but quite different behaviors in the $\gamma$ degree of freedom. In 86,88Se, both PES and c.w.f. are predicted to be extending in the $\gamma$ direction, in good agreement with the O(6) limit. However, going to 90,92,94Se, two distinct minima with similar absolute energies are predicted in the PESs, hinting a possible shape coexistence in these nuclei. By looking into the c.w.f., the yrast states (4\textsuperscript{+} not shown) in 86Se are predicted to be prolate deformed, while the low-lying 2\textsuperscript{+} state features a competing oblate shape. Conversely, 94Se exhibits an oblate yrast band and a prolate 2\textsuperscript{+} state. For 92Se, the calculation predicts a $\gamma$-soft ground state which evolves to a stabilized oblate shape for the 4\textsuperscript{+} state, while the 2\textsuperscript{+} state exhibits a prolate-$\gamma$-soft deformation mixing with an oblate configuration. Overall, when following the Se isotopic chain, the SCCM calculations show an intriguing shape transition: the yrast states evolve from prolate (90Se) to oblate (94Se) through a transitional $\gamma$-soft (92Se) shape, and the 2\textsuperscript{+} states, conversely to the yrast states, undergo an oblate-$\gamma$-soft-prolate transition. Quantitatively, from 92Se to 94Se, the theoretical calculated $E_\text{sp}$ evolves from $-42 \pm 55$ e fm\textsuperscript{2} for the 2\textsuperscript{+} state and from $-43 \pm 41$ e fm\textsuperscript{2} for the 2\textsuperscript{+} state. These transitions can be understood in the view of the O(6) limit in the IBA model: As pointed out in Refs. \cite{42,43}, the O(6) limit is the critical point of a first-order prolate-oblate shape-phase transition. The Se isotopes studied here were all
indicative of being located around the O(6) limit; however, it is difficult to be exactly at the critical point where the $\gamma$ degree of freedom has no global minimum. Finally, the calculations predicted intruder $0^+_2$ states strongly connected to the $2^+_2$ and $4^+_2$ states [see Fig. 3(b)], which have not been experimentally observed. The present measurement could be insensitive to those states due to their proximity to the $2^+_2$ states, resulting in weak $2^+_2 \rightarrow 0^+_1$ transition rates and branching ratios.

In summary, low-lying excited states in $^{88}$Se and for the first time in $^{90,92,94}$Se were studied. In total, 18 transitions from 13 newly proposed excitation levels were reported. A smooth, shallow drop of $E(2^+_1)$ was observed up to $N = 60$, yielding no evidence for a subshell closure at $N = 56$ nor a rapid onset of deformation at $N = 60$. It was found that all the proposed $2^+_2$ levels lie below or close to the $4^+_1$ states and their energies decrease continuously to below 1 MeV for $^{90,92}$Se, indicating triaxiality in these nuclei. The obtained results are in reasonable agreement with beyond-mean-field calculations based on the SCCM approach, which point out the important roles of the triaxial degree of freedom and shape coexistence for a proper description of the neutron-rich Se isotopes. Strikingly low-lying $0^+_2$ states were predicted in the same theoretical model. Finding these states is a key step forward in future experiments, while “safe” energy Coulomb excitation, though presumably beyond reach at presently existing facilities, may confirm the predicted deformed shapes in the Se isotopes.

We express our gratitude to the RIKEN Nishina Center accelerator staff for providing the stable and high-intensity uranium beam and to the BigRIPS team for operating the secondary beams. The development of MINOS and the core MINOS team were supported by the European Research Council through the ERC Grant No. MINOS-258567. This
work was supported by the IPA program at RIKEN Nishina Center, by GSI-Darmstadt computing facilities and Ministerio de Economía y Competitividad under Contract FIS-2014-53434 and Programa Ramón y Cajal 2012 No. 11420, by the German BMBF Grants No. 05P15PKFNA, No. 05P12RDFN8, and No. 05P15RDFN1, by Ministerio de Economy Competitividad under Contract FPA2014-57196-C5-4-P, by the Hundred-Talent Program at Chinese Academy of Sciences, by the National Natural Science Foundation of China under Grant No. 11405224, by the Japan Society for the Promotion of Science under Grant No. 2604327, and by the Vietnam Ministry of Science and Technology.